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The trouble with de Sitter space

Naureen Goheer
Department of Mathematics and Applied Mathematics, University of Cape Town
7701 Rondebosch, Cape Town, South Africa
E-mail: naureen@cosmology.mth.uct.ac.za

Matthew Kleban
Department of Physics, Stanford University
Stanford, CA 94305-4060, USA
E-mail: mkleban@stanford.edu

Leonard Susskind
Korea Institute for Advanced Study and Department of Physics, Stanford University
Stanford, CA 94305-4060, USA
E-mail: susskind@sewerrat.stanford.edu

Abstract: In this paper we assume the de Sitter space version of black hole Complementarity which states that a single causal patch of de Sitter space is described as an isolated finite temperature cavity bounded by a horizon which allows no loss of information. We discuss the how the symmetries of de Sitter spaceshould be implemented. Then we prove a no go theorem for implementing the symmetries if the entropy is finite. Thus we must either give up the finiteness of de Sitter spaceentropy or the exact symmetry of the classical space. Each has interesting implications for the very long time behavior. We argue that the lifetime of a de Sitter phase can not exceed the Poincaré recurrence time. This is supported by recent results of Kachru, Kallosh, Linde and Trivedi.

Keywords: Black Holes in String Theory, AdS-CFT and dS-CFT Correspondence, Space-Time Symmetries, Thermal Field Theory.

Contents

1. Thermofield dynamics and horizons

An exact formulation of the quantum theory of an accelerating universe appears to be very elusive [1]. Thus far the holographic principle [2, 3, 4] has not produced a dual description of de Sitter space analogous to the holographic duality between anti-de Sitter space and Super Yang Mills theory [5, 6, 7]

A number of authors [10]–[18] have argued for a de Sitter complementarity principle similar to the black hole version [8, 9]. While differing in the details, all versions agree that physics in a single causal patch of de Sitter space should be described as an isolated quantum system at at finite temperature, and that the thermal entropy of the system should be finite. From previous experience, especially with Matrix theory and the AdS/CFT duality, we can expect that the holographic dual of de Sitter space will not look much like classical relativity. Most likely its degrees of freedom will be very nonlocally connected to to the local semiclassical description of the space. That raises an interesting question: Suppose we were presented with the de Sitter space holographic dual. How would we recognize it as such? The answer for Matrix theory and AdS/CFT was initially through the symmetries, which exactly matched. Supersymmetry, galilean symmetry, and conformal invariance were especially important. Thus we might try to recognize the de Sitter dual by finding that its symmetries include the $\mathrm{SO}(d,1)$ symmetry of d-dimensional de Sitter space. However, most of the group (and the most interesting part of it) involve transformations which take the causal patch to another patch. The manifest symmetries of the causal patch are the $\mathrm{SO}(d-1)$ rotations which preserve the horizon, and the time translations. There are another $d-1$ compact generators and $d-1$ noncompact generators which displace the observer to a new patch. These generators do not act on the Hilbert space of a single observer. To understand how they act we have to introduce a bigger space called the thermofield double.

Thermofield theory was invented [22] in the context of many body theory for the purpose of simplifying the calculation of real time correlation functions in finite temperature systems. Its connection with black holes was realized by Israel [21] and elaborated in the holographic context by Maldacena [20]. Begin with a thermal system characterized by a Hilbert space $\mathcal{H}$, a hamiltonian H and a temperature $T = 1/\beta$. The thermal ensemble is described by the Bolzmann density matrix

$$\rho = \frac{1}{Z} \exp\left(-\beta H\right). \tag{1.1}$$

Thermal expectation values of real time correlation functions (response functions) contain information not only about equilibrium properties but also non–equilibrium dynamics. A typical response function can be thought of as determining the future behavior of a system that has been kicked out of equilibrium. A typical example has the form

$$\begin{aligned} \langle A(0)B^{\dagger}(t)\rangle &= \frac{1}{Z}\mathrm{Tr}e^{-\beta H}Ae^{iHt}Be^{-iHt} \\ &= \sum_{ij} A_{ij}B^{\dagger}_{ji}e^{i(E_i-E_j)t-\beta E_i}\,. \end{aligned} \tag{1.2}$$

In fact the full set of thermal real time correlators contains information about physics arbitrarily far from equilibrium.

The thermofield formalism introduces a fictitious system that includes two copies of the original system. The two copies are labeled 1 and 2. Copy 1 is thought of as the real system, while 2 is introduced as a trick. The doubled system is called the thermofield double. The Hilbert space for the thermofield double is $\mathcal{H}_{tf} = \mathcal{H}_1 \bigotimes \mathcal{H}_2$ where each factor space is identical to the original Hilbert space. The hamiltonian for the thermofield double is

$$H \equiv H_{tf} = H_1 - H_2\,. \tag{1.3}$$

Notice that the energy eigenvalues for subsystem 2 are of opposite sign from those of 1. We now construct the entangled state

$$|\psi\rangle = \frac{1}{Z^{\frac{1}{2}}}\sum_i e^{-\frac{1}{2}\beta E_i}|E_i, E_i\rangle\,, \tag{1.4}$$

where the state $|E_i, E_j\rangle$ means the eigenvector of H_1 and H_2 with eigenvalues E_i, E_j. The state $|\psi\rangle$ is a particular eigenvector of H_{tf} with eigenvalue zero. Obviously $|\psi\rangle$ has been constructed so that the density matrix for subsystem 1 is just the thermal density matrix ρ at temperature β.

Operators which belong to subsystem 1 have the form $A_1 \bigotimes I_2$, and will be called A_1. Operators associated with subsystem 2 will be defined in a similar manner except with an additional rule of hermitian conjugation:

$$A_2 \equiv I_1 \bigotimes A_2^{\dagger}\,. \tag{1.5}$$

The correlation function in 1.2 may be written as an expectation value:

$$\langle\psi|A_1(0)B_1^{\dagger}(t)|\psi\rangle\,. \tag{1.6}$$

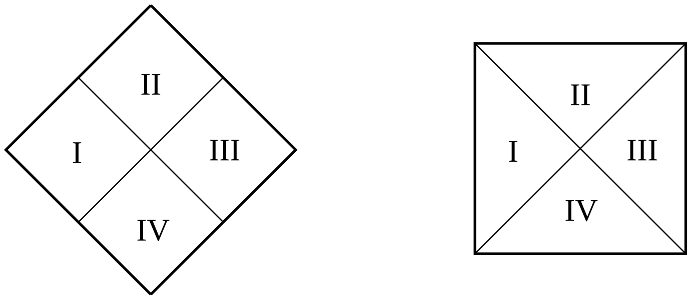

Figure 1: Penrose diagrams for Minkowski space (on the left) and de Sitter space or the eternal anti-de Sitter black hole (on the right).

Although no physical significance is ordinarily attached to correlators involving A_2, we may formally define them; for example

$$\langle\psi|A_1(0)B_2(t)|\psi\rangle\,. \tag{1.7}$$

Note that in both 1.6 and 1.7 the time dependence of the operators can be defined via the Heisenberg picture, using the full thermofield hamiltonian H_{tf}. Let us examine expression 1.7 more carefully. It can be written explicitly in the form

$$\langle\psi|A_1(0)B_2(t)|\psi\rangle = \sum_{ij} e^{-(\frac{\beta}{2}E_i+\frac{\beta}{2}E_j)} e^{i(E_i-E_j)t} A_{ij} B^{\dagger}_{ji}\,. \tag{1.8}$$

This is similar, but not identical, to 1.2. The similarity between the two can be made more apparent by writing 1.8 as

$$\begin{aligned}\langle\psi|A_1(0)B_2(t)|\psi\rangle &= \sum_{ij} e^{-\frac{\beta}{2}E_i} e^{i(E_i-E_j)(t-i\frac{\beta}{2})} A_{ij} B^{\dagger}_{ji}\\ &= \left\langle A(0)B\left(t-i\frac{\beta}{2}\right)\right\rangle.\end{aligned} \tag{1.9}$$

This demonstrates that correlators involving both copies can be expressed as analytic continuations of ordinary thermal correlators.

We will now consider the relationship between thermofield dynamics and quantum field theory in spaces with horizons. The simplest example involves Rindler space. One plus one dimensional Minkowski space can be divided into four quadrants; I, II, III and IV (see figure 1). Quadrants I and III consist of points separated from the origin by space–like separation, while II and IV are displaced in by timelike intervals. Quadrant I is Rindler space, and can be described by means of the metric

$$ds^2 = r^2dt^2 - dr^2\,, \tag{1.10}$$

where r is proper distance from the origin, and t is dimensionless Rindler time. The Rindler quadrant may be described by the Unruh thermal state with temperature

$$T_{\rm rind} = \frac{1}{2\pi} = \frac{1}{\beta_{\rm rind}}\,. \tag{1.11}$$

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Quadrant III is a copy of quadrant I, and can be precisely identified with the thermofield double. To see this we first of all note that the Rindler hamiltonian is the boost generator. Since the Minkowski vacuum is boost invariant, it is an eigenvector of the boost generator with vanishing eigenvalue. Furthermore, the Minkowski vacuum is an entangled state of the degrees of freedom in the two quadrants I and III. Finally, it is well known that when the density matrix for quadrant I is obtained by tracing over III the result is a thermal state at the Rindler temperature. It is easy to see that correlators between fields in quadrants I and III are related by exactly the same analytic continuations derived from thermofield dynamics. To see this, recall that the usual Minkowski variables X^0, X^1 are related to the Rindler coordinates by

$$\begin{aligned} X^0 &= r \sinh t \\ X^1 &= r \cosh t\,. \end{aligned} \tag{1.12}$$

Since the inverse Rindler temperature 1.11 is 2π, the continuation in equation 1.9 is

$$t \to t - i\pi\,, \tag{1.13}$$

or, from 1.12, $X^\mu \to -X^\mu$. Thus the thermofield continuation takes quadrant I to quadrant III.

The symmetry of Rindler space includes only the Rindler time translations. The choice of origin implicit in the identification of Rindler space breaks the symmetries of Minkowski translations. However, once it is realized that the Minkowski vacuum coincides with the thermofield double $|\psi\rangle$, it becomes clear that the action of the translations can be represented in the product Hilbert space $\mathcal{H}_{tf} = \mathcal{H}_1 \otimes \mathcal{H}_2$. The full Poincaré algebra in $1+1$ dimensions is

$$\begin{aligned} [P_0, H_{\mathrm{rind}}] &= iP_1 \\ [P_1, H_{\mathrm{rind}}] &= iP_0 \\ [P_0, P_1] &= 0. \end{aligned} \tag{1.14}$$

There is one discrete symmetry which acts on the thermofield double; namely, the parity operation $X^1 \to -X^1$. This transformation as well as the transformations generated by P^μ obviously mix the degrees of freedom of the thermofield double in a non trivial way. The lesson of this section is that the full symmetry of geometries with horizons acts on the thermofield double Hilbert space, and not on the space of available to an observer one one side of the horizon. Moreover the symmetry transformations mix the degrees of freedom of the two copies.

With this in mind let us consider the eternal AdS eternal black hole. Maldacena [20] has noted that the eternal black hole geometry has two boundaries, one in quadrant I of the Penrose diagram and one in quadrant III. Maldacena argues that the eternal black hole should be described by two copies of the usual boundary conformal field theory, and that the two copies are nothing but the thermofield double.[1] Moreover the state $|\psi\rangle$ is the Hartle Hawking state.

[1] An extremely interesting program is being pursued by Kraus, Ooguri and Shenker in which thermofield correlators are used to probe physics behind black hole horizons in quadrants II and IV.

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Typically there are symmetries which do not mix the two copies of the thermofield double and symmetries which do mix them. In the case of the eternal AdS black hole the symmetry which preserves the separate boundaries includes time translations and rotations of space. The thermal state describing a single copy is not an eigenvector of either the energy or the angular momentum operators of copy 1. However the Hartle Hawking thermofield double state ψ is an eigenvector with vanishing eigenvalue of H_{tf} and $J_{tf} \equiv J_1 + J_2$. As in the Rindler case there is also a discrete symmetry which interchanges the copies. The Hartle Hawking state is invariant under this transformation as well.

2. De Sitter space

De Sitter space is another example of a space with a horizon. Its Penrose diagram is identical to that of the AdS black hole. For simplicity we will consider the case of $2+1$ dimensional de Sitter space. The space can be globally described by the metric

$$ds^2 = R^2 \left(dt^2 - (\cosh t)^2 d\Omega_2^2\right) . \tag{2.1}$$

Quadrant I is now referred to as a causal patch of de Sitter space. Its metric is given by

$$\frac{ds^2}{R^2} = (1-r^2)dt^2 - (1-r^2)^{-1}dr^2 - r^2 d\theta^2 . \tag{2.2}$$

As in the black hole case, the properties of the causal patch are described by a thermal density matrix [19]. Unlike the AdS case we do not know the details of the holographic dual that describes the causal patch, but we will assume that such a description exists. The assumption that every causal patch has its own quantum mechanical description as an isolated system is the analog of Black Hole Complementarity (which Parikh, Savonije and Verlinde refer to as Observer Complementarity [18]). Once again we can introduce the thermofield formalism and introduce a copy of the causal patch representing quadrant III of the de Sitter Penrose diagram.

The symmetry group of d dimensional de Sitter space is the group SO$(d,1)$, which is simply the Lorentz group of the $d+1$ dimensional embedding space of the de Sitter hyperboloid. In the case of $d=3$, there are six generators: three boosts and three rotations. One of the rotations and one of the boosts preserves the causal patch; we will refer to these as J and H respectively. J generates spatial rotations ($\theta \to \theta + \text{const}$), and H generates time translations. More precisely H shifts time forward in quadrant I and backward in quadrant II. The remaining two boosts K_1, K_2 and rotations R_1, R_2 do not preserve the causal patch - that is, they mix the two copies of the thermofield double in a way analogous to the action of Minkowski translations on the Rindler wedges. The generators satisfy the algebra

$$\begin{aligned}
[R, J_i] &= i\epsilon_{ij} J_j \\
[J_i, J_j] &= i\epsilon_{ij} R \\
[H, J_i] &= i\epsilon_{ij} K_j \\
[H, K_i] &= -i\epsilon_{ij} J_j
\end{aligned}$$

$$
\begin{aligned}
[K_i, K_j] &= -i\epsilon_{ij} R \\
[J_i, K_j] &= i\epsilon_{ij} H \\
[H, R] &= 0\,.
\end{aligned} \tag{2.3}
$$

The thermofield formalism requires $H = H_{tf} = H_1 - H_2$, where H_1 acts on $\mathcal{H}_1$ and H_2 acts on $\mathcal{H}_2$ independently. The generators J_i rotate the static patch to a new patch; a rotation by π interchanges the thermofield double copies. Since the J_i mix the two copies, it is evident that they can not be expressed as a sum or difference of operators in $\mathcal{H}_1$ and $\mathcal{H}_2$. The generators K_i are hamiltonians for static patches which are rotated by a relative angle of $\pi/2$. They too mix the degrees of freedom in the original patch.

The de Sitter group is part of the group of coordinate transformations, which in general relativity is the gauge group. For this reason physical states should be invariant under the action of any of its generators. Note that this refers to the generators in the thermofield double and not to the individual copies. According to the definition 1.4, the generator $H_{tf} = H_1 - H_2$ annihilates $|\psi\rangle$.

$$
H_{tf}|\psi\rangle = (H_1 - H_2)\sum_i e^{-\frac{1}{2}\beta E_i}|E_i, E_i\rangle = 0\,. \tag{2.4}
$$

A de Sitter invariant state must also satisfy

$$
J|\psi\rangle = 0\,. \tag{2.5}
$$

This is a highly nontrivial condition but it is part of the definition of a quantum version of de Sitter space. Solutions are known to exist. The Hartle Hawking state for any quantum field theory in a de Sitter background satisfies 2.4 and 2.5. The condition that K annihilates ψ is of course a consequence of the commutation relations.

3. Finite entropy

There is one more condition that we must satisfy in order to have a quantum dual of de Sitter space; the thermal entropy of one causal patch is finite. The value of the entropy is connected to the size of the de Sitter space by the Gibbons Hawking [19] formula for de Sitter entropy:

$$
S = \frac{\text{Horizon Area}}{4G}. \tag{3.1}
$$

For our purposes we can regard 3.1 as defining the size of the de Sitter space through its entropy, which we can compute in terms of the density matrix 1.1:

$$
S = -\mathrm{Tr}(\rho \log \rho)\,. \tag{3.2}
$$

The only thing we will assume is that the entropy is finite. The consequences of that for the spectrum of H_1 are very clear: the eigenvalues E_i must be discrete. More precisely, the number of eigenvalues with energy less than any specified value must be finite. This condition implies that the spectrum of $(H_1 - H_2)$ must be countable.

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In the literature much stronger conditions have been assumed for the spectrum of H_1. Banks and Fischler have conjectured that the Hilbert space of states $\mathcal{H}_1$ should be finite dimensional. This may be so, but it does not follow from the finiteness of the entropy. The Entropy is only equal to the dimensionality of the space of states when the temperature is infinite. Entropy can certainly be finite even though the Hilbert space of states is infinite dimensional. We are assuming only the weaker condition that the spectrum is discrete.

We are now ready to prove a no–go theorem. The finiteness of the entropy is incompatible with the existence of the symmetry generators (H, J_i, K_j), and the requirement that H be a hermetian operator. First, define the hermitian operator $L \equiv J_1 + K_2$. It follows from 2.3 that

$$[H, L] = iL\,. \tag{3.3}$$

In general, L is not a good operator on the spectrum of H. If it were, 3.3 would imply that L acts as a raising operator on the spectrum of H but would raise the energy by i, which of course is inconsistent with the hermeticity of H. However, although the generator L itself is not bounded, $\exp(iL)$ is a group element and a good operator. Using 3.3, it is easy to see that

$$e^{iL}(t) \equiv e^{iHt}e^{iL}e^{-iHt} = e^{e^{iHt}iLe^{-iHt}} = e^{iLe^{-t}}\,. \tag{3.4}$$

We will now assume that the spectrum of H is countable, and use the assumption to derive a contradiction. We have

$$\left|\langle\alpha|e^{iL}|\alpha\rangle\right| = 1 - \delta\,. \tag{3.5}$$

Here $|\alpha\rangle$ is some state in the Hilbert space, and $\delta > 0$ because e^{iL} is unitary and L is non-zero. We also have

$$F(t) \equiv \langle\alpha|e^{iL(t)}|\alpha\rangle = \langle\alpha|e^{iHt}e^{iL}e^{-iHt}|\alpha\rangle = \langle\alpha|e^{iLe^{-t}}|\alpha\rangle\,. \tag{3.6}$$

From this, $F(t) \to 1$ as $t \to \infty$, and $F(0) = 1 - \delta < 1$. We will now prove that $F(t)$ is quasiperiodic (see e.g. the appendix of [17]).

Any sum of the form

$$\sum_{n=1}^{\infty} f_n e^{i\omega_n t} \tag{3.7}$$

is quasiperiodic if

$$\sum_{n=1}^{\infty} |f_n|^2 < \infty. \tag{3.8}$$

Therefore, it suffices to show that $F(t)$ can be written as a sum of this form. But, expanding the state $|\alpha\rangle$ in the energy basis,

$$F(t) = \sum_{n,m} f_n^* f_m \langle n|e^{iL}|m\rangle e^{i(\omega_n - \omega_m)t}\,. \tag{3.9}$$

Consider the sum

$$\sum_{m,n} f_n^* f_m f_m^* f_n \langle n|e^{iL}|m\rangle\langle m|e^{-iL}|n\rangle = \sum_n |f_n|^2 \sum_m |f_m|^2 \langle n|e^{iL}|m\rangle\langle m|e^{-iL}|n\rangle\,. \tag{3.10}$$

Considering the inner sum, we have (since $\sum_m \langle n|e^{iL}|m\rangle\langle m|e^{-iL}|n\rangle = 1$, and the terms are real and positive)

$$\sum_m |f_m|^2 \langle n|e^{iL}|m\rangle\langle m|e^{-iL}|n\rangle \leq 1\,, \tag{3.11}$$

and therefore

$$\sum_{m,n} f_n^* f_m f_m^* f_n \langle n|e^{iL}|m\rangle\langle m|e^{-iL}|n\rangle \leq 1. \tag{3.12}$$

This shows that $F(t)$ satisfies the criterion 3.8, and hence $F(t)$ is quasiperiodic. Therefore, since $F(0) < 1$, $F(t)$ can not tend to 1 as $t \to \infty$, and we have a contradiction

This proves that H can not have a countable spectrum. and therefore that the E_i cannot be discrete and the entropy cannot be finite.

Shenker [24] has suggested an interesting viewpoint; that of anomalies. It is fairly clear that in ordinary perturbation theory in de Sitter space the symmetry of the space will never break down. Furthermore, in every order the entanglement entropy of I and III will be infinite. This suggests that the same nonperturbative effects which make the spectrum discrete and the entropy finite, also break the de Sitter symmetry. A tantalizing hint is that the size of such a nonperturbative effect would scale as $\exp(-\text{area}/4G_N) \sim \exp(-S)$, which is the characteristic energy gap δE for a system with entropy S.

The same argument can be applied to Rindler space. Using 1.14 and replacing the operator L by $P_1 + P_0$ the spectrum H_{rind} is also proved to be all real numbers. Thus the entropy of Rindler space must be infinite. Obviously if the number of non–compact directions of spacetime is 3 or more, the horizon is infinite and the total entropy does diverge. It is only in the case of $1+1$ non–compact directions that the entropy could possibly be finite and conflict with Rindler symmetry. However, at least in the case of string theory it is known that compactification to $1+1$ dimensions can not yield a theory with translation invariance [23]. In the appendix we describe as an example the action of a Minkowski space translation on the Rindler thermofield double.

4. Conclusions

Finiteness of entropy appears to be incompatible with de Sitter symmetry. What are we to make of this no–go theorem? One possibility is that de Sitter space has infinite entropy. Perhaps only entropy differences are finite. However, this seems to run against the grain of everything we have learnt over the last decade. The horizon of de Sitter space is locally identical to that of a Schwarzschild black hole. It is hard to see why one would have an infinite entropy and the other not [26].

If the entropy is finite then the symmetry of different causal patches must be broken, the hamiltonians and energy spectra differing at least slightly. How would these effects manifest themselves? The discreteness of the energy spectrum introduces a new time scale of order $1/\delta$, where δ is of order the typical spacing of levels. For a system with entropy S the level spacing is of order $\exp(-S)$ and the time scale is the Poincaré recurrence [15, 16, 17] time $t_p \sim \exp S$. This is the time scale on which the discreteness of the spectrum causes significant effects. Thus the violations of de Sitter symmetry should become important for

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times of order t_p and recurrences should not respect the symmetry. We may speculate that no space can behave like idealized de Sitter space for times longer than t_p. For example, it is possible that eternal de Sitter simply does not exist, because there are always instabilities which cause the space to decay in a time shorter than the recurrence time.[2]

An alternative has been proposed by Bousso [4], which is simply that because any observer will be destroyed by thermal particles long before t_p, times longer than t_p should be regarded as having no operational meaning. In the same spirit, Banks, Fischler, and Paban [13] speculate that due to the physical constraints on the measuring process in de Sitter space , no experiment can ever last a time comparable to the recurrence time. Hamiltonians that agree for time scales less than $\sim t_p$ should be considered physically equivalent. In terms of the energy levels E_i this would mean that the level shifts going from one patch to another should be no bigger than the level spacing $\exp(-S)$. If this can be made precise and the physical consequences of different hamiltonians are identical, then the exact choice of hamiltonian would become part of the gauge redundancy in a single patch. The transformation from one patch to another would include such a gauge transformation. It is not clear to us whether this would mean that Poincaré recurrences are meaningless or if it means that there is an additional degree of unpredictability in their occurence.

5. Note added in revised version

We have argued on general grounds that de Sitter behavior can not persist for times of order the Poincaré recurrence time. In this note we will present an argument based on work by Kachru, Kallosh, Linde and Trivedi [27] that strongly supports this claim in the context of string theory.[3] Since string theory has supersymmetric ground states with vanishing vacuum energy it is possible to tunnel out of any positive local minimum of the scalar potential. Typically there will be a barrier to the tunneling which may be as high as the string or Planck scale. Kachru, Kallosh, Linde and Trevedi compute the rate for tunelling over such a barrier and find that it is always less than the recurrence time. The essence of the calculation is simple and can be understood by the following (over)simplified argument:

Obviously it is sufficient to calculate the rate for a fluctuation that takes the static patch to the top of the potential barrier since once that configuration is achieved, there is no further obstacle to rolling down to the vacuum with vanishing energy. Suppose the vacuum energy density (in Planck Units) at the top of the barrier is $\lambda < 1$. If the barrier is broad and fairly flat, we will have a temporary de Sitter phase with horizon area $\sim 1/\lambda$ and similar entropy. Now consider the rate for a thermal fluctuation to take take the static patch from the local minimum to the top of the potential. The standard formula for the rate of thermal fluctuations γ is

$$\gamma \ \exp(S_1 - S_0)\,.$$

[2]Some time ago, Simeon Hellerman [25] suggested that there may be a general bound coming from an acceleration–duration relation, which would prohibit arbitrarily long-lived accelerating spacetimes such as de Sitter.

[3]We are especially grateful to Andrei Linde for a very enlightening discussion of this point and for communicating the results of [27] to us.

where S_0 and S_1 are the entropies of the equilibrium phase and the fluctuation respectively. Since the recurrence time is $\exp S_0$ it is clear that the rate of thermal tunelling is always bigger than the rate of Poincaré recurrences. Thus is appears that in string theory, a metastable de Sitter phase can not live for a recurrence time.

This instability has implications for the energy spectrum of the metastable de Sitter. Since the lifetime is always shorter than the recurrence time, each energy level must develop a width γ at least of order $\exp -S$. In a typical situation of the type studied in [27], the width of each level is vastly greater than the level spacing, which is reminiscent of a Schwarzschild black hole in flat space.

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A. Rindler space

In this appendix we illustrate how the group of Minkowski translations acts on the Rindler degrees of freedom. For simplicity we study a 1+1 Minkowski toy model with a free scalar field ζ of mass m. The Minkowski coordinates are denoted by T, X and take values in $[-\infty, \infty]$. The lagrangian density for a free scalar field of mass m is given by

$$\mathcal{L} = \sqrt{-g}\left[g_{\mu\nu}\partial^\mu\zeta\partial^\nu\zeta - m^2\zeta\right], \tag{A.1}$$

Rindler coordinates are defined by

$$r = \sqrt{X^2 - T^2}, \qquad \tau = \frac{1}{2}\ln\frac{X+T}{X-T}. \tag{A.2}$$

They are defined on both the right and the left Rindler wedge, but time and radius run in opposite directions in the two wedges. In these coordinates the wave equation takes the form

$$\ddot{\zeta} - \frac{\partial^2}{\partial u^2}\zeta + m^2 e^{2u}\zeta = 0 \tag{A.3}$$

where $u = \ln r$ and dot indicates derivative with respect to τ. Let us note first that as we discussed earlier, the algebra of the Rindler hamiltonian and the Minkowski translations can only be consistent if the entropy of Rindler space is infinite. Since the u axis is unbounded at both ends it is not surprising that the entropy of the scalar field at finite temperature is infinite. However the important infinity comes from one end as can be seen from the wave equation (A.3): the mass term acts like a wall preventing the field from penetrating to large positive u. On the other hand the region of large negative u is still massless. In this case the divergent entropy comes from the $u \ll 0$. This is of course the region very near the horizon.

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Any solution of the wave equation can be expanded in terms of the basis functions

$$f_k = \frac{1}{\sqrt{c_k}} e^{iku - i\omega_k \tau} = \frac{1}{\sqrt{c_k}} r^{ik} e^{-i\omega_k \tau} \tag{A.4}$$

where $c_k = 4\pi\omega_k$ are normalization constants and $\omega_k = +\sqrt{k^2 + r^2 m^2}$.

The two thermofield copies of ζ at $\tau = 0$ are just the fields on the two half spaces $X > 0$ and $X < 0$. Each copy can be expanded in creation and annihilation operators for Rindler excitations. Denoting the operators by $a, a^\dagger$ and $b, b^\dagger$ for $X > 0$ and $X < 0$ respectively ζ can be expressed as

$$\begin{aligned}\zeta(r,\tau) \;=\; & \int \frac{dk}{\sqrt{c_k}} \left[r^{ik} e^{-i\omega_k \tau} a(k) + r^{-ik} e^{i\omega_k \tau} a^\dagger(k) \right] \Theta(X) + \\ & + \int \frac{dk}{\sqrt{c_k}} \left[r^{ik} e^{-i\omega_k \tau} b(k) + r^{-ik} e^{i\omega_k \tau} b^\dagger(k) \right] \Theta(-X)\,. \end{aligned} \tag{A.5}$$

The first integral corresponds to the part of the wave equation in the right Rindler wedge while the second integral corresponds to the part in the left Rindler wedge.

To illustrate the action of the Minkowski translations on the Rindler degrees of freedom we consider a translation along the X axis by distance a. The Rindler coordinates transform as

$$r' = \sqrt{(X-a)^2 - T^2} = \sqrt{r^2 + a^2 \mp 2ar\cosh\tau} \tag{A.6}$$

and

$$\tau' = \frac{1}{2} \ln \frac{X - a + T}{X - a - T} = \frac{1}{2} \ln \frac{\pm r\cosh\tau + r\sinh\tau - a}{\pm r\cosh\tau - r\sinh\tau - a}\,. \tag{A.7}$$

The upper signs apply in the right Rindler wedge ($X > 0$), the lower ones in the left Rindler wedge ($X < 0$).

In the right hand wedge, the shifted fields $\zeta, \dot{\zeta}$ at $\tau = 0$ have the form

$$\begin{aligned}\zeta(r) \;=\; & \int \frac{dk}{\sqrt{c_k}} [(r-a)^{ik} a(k) + (r-a)^{-ik} a^\dagger(k)] \Theta(r-a) + \\ & + \int \frac{dk}{\sqrt{c_k}} [(a-r)^{ik} b(k) + (a-r)^{-ik} b^\dagger(k)] \Theta(a-r) \\ \dot{\zeta}(r) \;=\; & -i \int \frac{dk}{\sqrt{c_k}} \omega_k r [(r-a)^{ik-1} a(k) - (r-a)^{-ik-1} a^\dagger(k)] \Theta(r-a) - \\ & -i \int \frac{dk}{\sqrt{c_k}} \omega_k r [(a-r)^{ik-1} b(k) - (a-r)^{-ik-1} b^\dagger(k)] \Theta(a-r)\,. \end{aligned} \tag{A.8}$$

The shifted field can also be to expanded in terms of new (shifted) creation and annihilation operators $c^\dagger$ and c. Concentrating on the right hand Rindler wedge:

$$\zeta(r,\tau)\mid_{\tau=0} = \int dk [f_k c(k) + f_k^* c^\dagger(k)] \tag{A.9}$$

where the basis functions f_k are defined as above.

The Bogoliubov transformation giving the operators $c, c^\dagger$ is easily obtained. For example we find

$$c^\dagger(l) = \int dk \left[D(l,k) a^\dagger(k) + E(l,k) a(k) + F(l,k) b^\dagger(k) + G(l,k) b(k) \right] \tag{A.10}$$

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with the Bogoliubov coefficients given by

$$\begin{aligned}
D(l,k) &= \frac{1}{4\pi\sqrt{\omega_k\omega_l}}(\omega_k + \frac{k}{l}\ \omega_l)a^{il-ik}B(-il+ik,-ik) \\
E(l,k) &= -\frac{1}{4\pi\sqrt{\omega_k\omega_l}}(\omega_k + \frac{k}{l}\ \omega_l)a^{il+ik}B(-il-ik,ik) \\
F(l,k) &= \frac{1}{4\pi\sqrt{\omega_k\omega_l}}\frac{l\omega_k - k\omega_l}{l-k}a^{il-ik}B(il,-ik) \\
G(l,k) &= -\frac{1}{4\pi\sqrt{\omega_k\omega_l}}\frac{l\omega_k - k\omega_l}{l+k}a^{il+ik}B(il,ik)\,,
\end{aligned} \tag{A.11}$$

where $B(a,b)$ is the Euler β–function:

$$B(a,b) = \int_0^1 y^{a-1}(1-y)^{b-1}dy \tag{A.12}$$

and we have used the identity $B(a,b+1) = \frac{b}{a+b}B(a,b)$. In the massless limit these coefficients simplify to give

$$\begin{aligned}
D(l,k) &= \frac{1}{2\pi}\sqrt{\frac{k}{l}}a^{il-ik}B(-il+ik,-ik)\ \Theta(kl) \\
E(l,k) &= -\frac{1}{2\pi}\sqrt{\frac{k}{l}}a^{il+ik}B(-il-ik,ik)\ \Theta(kl) \\
F(l,k) &= (sgn(l)-sgn(k))\frac{\sqrt{|kl|}}{4\pi(l-k)}a^{il-ik}B(il,-ik) \\
G(l,k) &= -(sgn(l)-sgn(k))\frac{\sqrt{|kl|}}{4\pi(l+k)}a^{il+ik}B(il,ik)\,.
\end{aligned} \tag{A.13}$$

Similar equations determine the transformed operators for the left wedge of Rindler space. In this way we explicitly see how the symmetries of Minkowski space mix the two copies of the thermofield double.

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